On radiative muon capture in hydrogen

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Abstract

We analyze the radiative capture of the negative muon in hydrogen using amplitudes derived within the chiral Lagrangian approach. Besides the leading order terms, given by the well-known Rood-Tolhoek Hamiltonian, we extract from these amplitudes the corrections of the next order in $1/M$ (M is the nucleon mass). In addition, we estimate within the same formalism also the $\Delta(1232)$ excitation effects and processes described by an anomalous Lagrangian. Using values of the parameters of the model obtained from the analysis of the pion photoproduction, which restricts the arbitrariness of the $\pi N\Delta$ and $\gamma N\Delta$ vertices, we find that the model can explain up to half of the discrepancy between the PCAC value g_P^{PCAC} of the induced pseudoscalar constant g_P and of g_P extracted from the recent TRIUMF experiment. If these parameters are allowed to vary independently, the model can describe the high energy part of the experimental photon spectrum reasonably well for the values of $g_P \approx g_P^{PCAC}$.

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1 Introduction

It is well known [1] that the charged weak interaction of the nucleon with a lepton is described by the weak hadron current,

$$J^a_{W,\mu}(q_1) \;=\; J^a_{V,\mu}(q_1) \;+\; J^a_{A,\mu}(q_1)\,, \tag{1}$$

where the vector part is given by the matrix element of the isovector Lorentz 4–vector current operator between the nucleon states,

$$\hat{J}^a_{V,\mu}(q_1) = i\left(g_V(q_1^2)\gamma_\mu - \frac{g_M(q_1^2)}{2M}\sigma_{\mu\,\nu}q_{1\,\nu}\right)\frac{\tau^a}{2}\,, \tag{2}$$

and the axial–vector part is analogously,

$$\hat{J}^a_{A,\mu}(q_1) = i\left(-g_A(q_1^2)\gamma_\mu\gamma_5 + i\frac{g_P(q_1^2)}{m_l}q_{1\,\mu}\gamma_5\right)\frac{\tau^a}{2}\,. \tag{3}$$

Here a is the isospin index, m_l is the lepton mass and the 4–momentum transfer is given by $q_{1\,\mu} = p'_\mu - p_\mu$, where p'_μ (p_μ) is the 4–momentum of the final (initial) nucleon.

The least known of the four form factors entering the currents Eqs. (2) and (3) is the induced pseudoscalar coupling constant, $g_P(q_1^2)$, in the axial–vector current $\hat{J}^a_{A,\mu}$. Actually, its presence in the axial–vector current (3) tests our understanding of the basic strong and weak interaction processes, such as the strong πNN vertex and the weak pion decay. Elementary calculations lead to

$$g_P(q_1^2) = 2Mg_A m_l\,\Delta^\pi_F(q_1^2)\,, \tag{4}$$

where $\Delta^\pi_F(q_1^2)$ is the pion propagator.

The matrix element of the axial current $\hat{J}^a_{A,\mu}$ should satisfy the PCAC equation. It is easy to verify that

$$\bar{u}(p')q_{1\,\mu}\hat{J}^a_{A,\mu}u(p) = \bar{u}(p')\left[2Mg_AF_A(q_1^2) - \frac{g_P(q_1^2)}{m_l}q_1^2\right]\gamma_5\frac{\tau^a}{2}u(p)\,. \tag{5}$$

It is seen from this equation, that if

$$\tilde{g}_P(q_1^2) = 2Mg_Am_l\frac{1-F_A(q_1^2)}{q_1^2}\,, \tag{6}$$

is subtracted from $g_P(q_1^2)$, then indeed, the partial conservation of the axial–vector current is valid. Here we put

$$g_A(q_1^2) = g_AF_A(q_1^2)\,, \tag{7}$$

with $g_A \equiv g_A(0) = -1.26$.

The best way to search for the effect of the form factor $g_P(q_1^2)$ is the muon capture. In the elementary process of the ordinary muon capture (OMC),

$$\mu^- + p \longrightarrow \nu_\mu + n\,, \tag{8}$$

the value of the induced pseudoscalar coupling constant (4) is predicted by PCAC to be,

$$g_P^{PCAC}(q_1^2 = 0.877 m_\mu^2) = \frac{2Mm_\mu}{0.877 m_\mu^2 + m_\pi^2}\, g_A = 6.78\, g_A\,, \tag{9}$$

while for the $\tilde{g}_P(q_1^2)$ in the chiral model [2] [3] we have,

$$\tilde{g}_P^{PCAC}(q_1^2 = 0.877 m_\mu^2) = \frac{2Mm_\mu}{0.877 m_\mu^2 + m_{a_1}^2}\, g_A = 0.12\, g_A\,, \tag{10}$$

where m_{a_1} is the mass of the axial vector meson $a_1(1260)$. So the correction to the $g_P(q_1^2)$ from other than one–pion exchange mechanisms presumably can be expected to be quite small.

However, a very flat dependence of the transition rate on g_P in the OMC by proton (8) provides its world average value [4] with an error of ≈ 20 % and particular experiments have an error larger by a factor of ≈ 2.

Recently, a very precise experimental study of the muon capture by ${}^3He$ [5],

$$\mu^- \,+\, {}^3He \,\longrightarrow\, \nu_\mu \,+\, {}^3H\,, \tag{11}$$

yields the transition rate

$$\Gamma_{exp} \,=\, 1494 \,\pm\, 4\, s^{-1}\,, \tag{12}$$

which allowed [6] an extraction of the value of g_P with an accuracy of ≈ 20 % from this experiment alone,

$$\frac{g_P}{g_P^{PCAC}} \,=\, 1.05 \,\pm\, 0.19\,, \tag{13}$$

where for the reaction (11)

$$g_P^{PCAC}(q_1^2 = 0.954\, m_\mu^2) \,=\, 6.59\, g_A\,. \tag{14}$$

A contribution of 20 % due to the meson exchange current effect turned out to be essential to get the calculated transition rate,

$$\Gamma_{th} \,=\, 1502 \,\pm\, 32\, s^{-1}\,, \tag{15}$$

into agreement with the data (12). Let us note that a further improvement of the extracted value of g_P is hindered by an uncertainty of ≈ 2 % in calculations [6] which will be difficult to improve. The main uncertainty arises from the less known parameters of the Δ excitation processes.

Another attractive tool to extract the value of g_P is the radiative muon capture (RMC). It is observed [7] that in the hadron radiative processes contributing to the amplitude of the elementary reaction,

$$\mu^- + p \longrightarrow \nu_\mu + \gamma + n \,, \tag{16}$$

the muon propagator entering Eq. (4) is larger by a factor of ≈ 3 than in OMC (8) at larger values of the photon momentum k. The theory of the RMC was elaborated by many authors during the decades (see Refs. [7]–[19] and references therein).

The nuclear Hamiltonian suitable for use in nuclear physics calculations was provided by Rood and Tolhoek [12]. It contains the leading order terms derived from the conserved RMC amplitude given by a set of the Feynman diagrams. Christillin and Servadio [15] rederived in an elegant way the RMC amplitude obtained earlier by Adler and Dothan [13] using the low energy theorems. This amplitude consists, besides the leading order terms, of the next to leading order terms in the $1/M$ expansion. They have also found that higher order terms cannot be obtained using this method. Recently, this amplitude was produced [3] from a chiral Lagrangian of the $N\pi\rho\omega a_1$ system. It satisfies the corresponding continuity equations and the consistency condition exactly. Higher order terms follow without any restriction. It was shown that the leading terms coincide with those given by the low energy theorems. However, the next to leading order terms differ, which is given by a different prescription to pass towards higher energies.

The above mentioned set of the relativistic Feynman diagrams was used by Fearing [16] to calculate the photon energy spectrum for the reaction (16). This work was later extended by Beder and Fearing [20] by considering also the contribution from the Δ excitation processes. A recent comparison of the TRIUMF high precision experiment [21],[22] with the Beder–Fearing calculations provided a value of g_P which is enhanced by ≈ 50 % in comparison with the PCAC prediction (9). Later attempts [23]–[25] within the framework of the chiral perturbation theory did not change much.

This situation makes the expectation of the result from the next TRIUMF high precision experiment in helium,

$$\mu^- + {}^3He \longrightarrow \nu_\mu + \gamma + {}^3H \,, \tag{17}$$

with a particular tension. However, one should have in mind also complications analogous to those in reaction (11) and non-negligible meson exchange current effects are to be expected, which makes the analysis much more difficult. It is clear that the Beder–Fearing relativistic formalism is not applicable in calculations with the realistic 3N wave functions and a consistent non–relativistic approach should be developed. Here we make an independent step in this direction by making the non–relativistic reduction of the amplitudes derived in [3] from a chiral Lagrangian of the $N\pi\rho\omega a_1$ system. As a result, we get an effective Hamiltonian which is in the leading order in $1/M$ close to the one obtained by Rood and Tolhoek [12] but not identical with it. We also apply the constructed effective Hamiltonian to compute the photon energy spectra for the reaction (16) in the energy interval 60 MeV–99 MeV and various spin states. The results stated below concern the mixture of muonic states relevant to the TRIUMF experiment [21],[22] (6.1 % in the singlet μp state, 85.4 % in the ortho pμp state and 8.5 % in the para pμp state). Our reduction provides also the next to leading order terms, a part of which follows from the standard amplitudes used already by Rood and Tolhoek and Fearing. Added to the leading order terms, they should reproduce with a good accuracy results given in [16] *. At the photon energy interval 60 MeV–99 MeV, these corrections suppress the leading order terms by 1–9 %. Another part of the next to leading order terms is produced by reduction of additional relativistic amplitudes following from our chiral Lagrangian. It turns out to compensate the damping effect of the first group of corrections by 2–4 %. We shall call it hard pion (HP) correction.

Next we include the Δ isobar using again the formalism of chiral Lagrangians developed in [2],[27], which we extend by adopting results of Refs. [28]–[30]. Then the resulting $N\Delta\pi\rho a_1$ Lagrangian consists of three terms and is characterised by three couplings and four arbitrary parameters A, X, Y, Z. In its turn, each term contains a tensor of the form

$$\Theta_{\mu\nu}(B) = \delta_{\mu\nu} + \left[\frac{1}{2}(1 + 4B)A + B\right]\gamma_\mu\gamma_\nu\,, \quad B = X, Y, Z\,, \tag{18}$$

which ensures the independence of the Δ contribution to the S–matrix on the parameter A. The choice $A = -1$ simplifies the Δ propagator considerably. The parameters X, Y, Z, which reflect the off–shell ambiguity of the massive spin 3/2 field were found [28]–[32] by analyzing the data on pion photoproduction †. The value of these parameters depends on how the pion production amplitude is unitarized. This model does not require the use of the Breit–Wigner form of the Δ propagator.

Let us note that besides the adopted model [28]–[30], other models [34],[35] were developed to describe the production of pions on protons by the electromagnetic interaction. All these models consider the same non–resonant Lagrangian of the $N\pi\rho\omega$ system, but differ principally in the treatment of the Δ isobar and in the method of unitarization of the πN amplitude.

In the calculations of the Δ excitation effect in reaction (16), Beder and Fearing [20] considered a model for needed vertices with $\Theta_{\mu\nu} = \delta_{\mu\nu}$ and the Breit-Wigner form of the Δ propagator. On the other side, they took the needed $\gamma N\Delta$ coupling from [29], thus introducing an inconsistency into calculations. This model provides about 7% effect from the Δ excitation to the photon spectrum.

We have also analyzed the contribution due to amplitudes constructed from an anomalous Lagrangian of the $\pi\rho\omega a_1$ system [36],[37]. We have found that the influence of this contribution on the photon energy spectrum is negligeable.

In order to compare our form factors with the results of Ref. [12], we define in Sect. 2 the effective Hamiltonian analogously and we consider the velocity independent part only. Then

*These corrections up to the order $1/M^2$ were discussed in [26].

†This model describes well also the latest data on the π^0 electroproduction on the proton [33].

in Sect. 3, we present the results for the form factors g_i in the leading– and in the next to leading orders in $1/M$, which follow from our amplitudes [3]. Further we deal with the contribution to g_i's from the Δ excitation amplitudes of our model and we compare our effective weak $N\Delta$ vertex with the one used in [20]. Finally, we discuss briefly the RMC amplitudes stemming from the anomalous Lagrangian.

In Sect. 4, we present our numerical results for the high energy part of the photon spectra for reaction (16) and we give our conclusions. Our main result is an enhancement up to $\approx$ 7–15 % relative to the calculations without including the Δ isobar and the HP correction, which can explain up to half of the discrepancy between the PCAC value of g_P and of g_P extracted from the experiment [21],[22], if we take the values of the parameters found from the data on pion photoproduction [29]–[32]. If we allow to change the parameters of the model independently, we can generate for the values of $g_P \approx g_P^{PCAC}$ the photon spectra, which are close to the one which should correspond to the experimental photon spectrum.

2 Effective Hamiltonian for RMC

In presenting the effective Hamiltonian, we follow Rood and Tolhoek [12]. Then the velocity independent part is,

$$
\begin{aligned}
H^{(0)}_{eff} \;=\; & \frac{1}{\sqrt{2}m_\mu}\left(1-\vec{\sigma}_l\cdot\hat{\nu}\right)\left[g_1\left(\vec{\sigma}_l\cdot\vec{\varepsilon}\right)+g_2\left(\vec{\sigma}\cdot\vec{\varepsilon}\right)+g_3 i\left(\vec{\sigma}\cdot\vec{\varepsilon}\times\vec{\sigma}_l\right)\right.\\
& +g'_4\left(\vec{\sigma}_l\cdot\vec{\varepsilon}\right)\left(\vec{\sigma}\cdot\hat{k}\right)+g''_4\left(\vec{\sigma}_l\cdot\vec{\varepsilon}\right)\left(\vec{\sigma}\cdot\hat{\nu}\right)+g'_5\left(\vec{\sigma}_l\cdot\hat{k}\right)\left(\vec{\varepsilon}\cdot\hat{\nu}\right)+g'_6\left(\vec{\varepsilon}\cdot\hat{\nu}\right)\\
& +g'_7 i\left(\vec{\sigma}\cdot\hat{k}\times\vec{\varepsilon}\right)+g''_7 i\left(\vec{\sigma}\cdot\hat{\nu}\times\vec{\varepsilon}\right)+g'_8\left(\vec{\sigma}_l\cdot\hat{k}\right)\left(\vec{\sigma}\cdot\vec{\varepsilon}\right)+g''_8\left(\vec{\sigma}_l\cdot\hat{\nu}\right)\left(\vec{\sigma}\cdot\vec{\varepsilon}\right)\\
& +g'_9\left(\vec{\sigma}_l\cdot\vec{\sigma}\right)+g'_{10}\left(\vec{\sigma}\cdot\hat{k}\right)\left(\vec{\varepsilon}\cdot\hat{\nu}\right)+g''_{10}\left(\vec{\sigma}\cdot\hat{\nu}\right)\left(\vec{\varepsilon}\cdot\hat{\nu}\right)\\
& \left.+g'_{11}\left(\vec{\sigma}_l\cdot\hat{k}\right)\left(\vec{\sigma}\cdot\hat{k}\right)\left(\vec{\varepsilon}\cdot\hat{\nu}\right)+g''_{11}\left(\vec{\sigma}_l\cdot\hat{k}\right)\left(\vec{\sigma}\cdot\hat{\nu}\right)\left(\vec{\varepsilon}\cdot\hat{\nu}\right)\right]\,. \qquad (19)
\end{aligned}
$$

Here $\vec{\sigma}_l$ ($\vec{\sigma}$) are the lepton (nucleon) spin Pauli matrices and $\hat{\nu}$ ($\hat{k}$) is the unit vector in the direction of the neutrino (photon) momentum vector $\vec{\nu}$ ($\vec{k}$). Not all the form factors are independent. Using equation,

$$
\vec{\varepsilon}_\lambda \;=\; -i\lambda\left(\hat{k}\times\vec{\varepsilon}_\lambda\right)\,, \qquad \vec{\varepsilon}_\lambda \;=\; \frac{1}{\sqrt{2}}\left(\vec{i}\,-\,\lambda\vec{j}\right)\,, \qquad (20)
$$

one gets redefinitions,

$$
g_2 \;\to\; g_2-\lambda\left(g'_7+yg''_7\right)\,, \quad g'_{10}\;\to\; g'_{10}+\lambda g''_7\,, \quad g'_8\;\to\; g'_8+\lambda g_3\,, \quad g'_4\;\to\; g'_4-\lambda g_3\,, \qquad (21)
$$

where $y=\left(\hat{\nu}\cdot\hat{k}\right)$. The last two terms in (19) are new in comparison with [12].

3 Contribution to $H_{eff}^{(0)}$ from the amplitudes of the chiral Lagrangian of the $N\Delta\pi\rho\omega a_1$ system

Here we discuss our amplitudes and contributions to g_i's. We start by presenting briefly the part of the RMC amplitude derived earlier [3] without Δ's (see Fig. 1), referring for details to Sect. 3 of that paper. Then we deal with the amplitudes describing the Δ excitation processes and we compare our effective vertices with those of Ref. [20]. Finally, we discuss the amplitudes stemming from the anomalous Lagrangian.

3.1 The RMC amplitude without Δ's

Besides the muon radiative part $M^a(k,q)$, the amplitude $T^a(k,q)$ [3] consists of three terms representing the hadron radiative amplitude,

$$T^a(k,q) = \frac{eG}{\sqrt{2}} \left\{ M^a(k,q) \,+\, l_\mu(0)\epsilon_\nu(k) \left[M^{B,\,a}_{\mu\nu}(k,q) \,+\, M^a_{\mu\nu}(\pi;k,q) \,+\, M^a_{\mu\nu}(a_1;k,q) \right] \right\} , \tag{22}$$

The amplitude $M^{B,\,a}_{\mu\nu}(k,q)$ consists of the nucleon Born terms (figs. 1a and 1b) and of some related contact amplitudes (figs. 1c and 1d). The amplitude $M^a_{\mu\nu}(\pi;k,q)$ contains the mesonic amplitude $M^{m.\,c.\,,a}_{\mu\nu}(\pi;k,q)$ (fig. 1e with B=π) and all contact terms where the electroweak vertex is connected with the nucleon by the pion (fig. 1f with B=π). The amplitude $M^a_{\mu\nu}(a_1;k,q)$ has graphically a similar structure as the amplitude $M^a_{\mu\nu}(\pi;k,q)$. These amplitudes satisfy separately continuity equations when contracted with the four momentum transfer q_μ of the weak vertex.

The sum of these amplitudes satisfies exactly the following Ward–Takahashi identities,

$$q_\mu \left[M^{B,\,a}_{\mu\nu} \,+\, M^a_{\mu\nu}(\pi) \,+\, M^{c.\,t.,\,a}_{\mu\nu}(a_1)\right] \,=\, i f_\pi m_\pi^2 \Delta^\pi_F(q)\, \mathcal{M}^a_{\pi,\,\nu} \,+\, i\varepsilon^{3\,a\,b}\bar{u}(p')\,\hat{J}^b_{W,\,\nu}(q_1)\,u(p)\,, \tag{23}$$

$$k_\nu \left[M^{B,\,a}_{\mu\nu} \,+\, M^a_{\mu\nu}(\pi) \,+\, M^{c.\,t.,\,a}_{\mu\nu}(a_1)\right] \,=\, i\varepsilon^{3\,a\,b}\bar{u}(p')\,\hat{J}^b_{W,\,\mu}(q_1)\,u(p)\,. \tag{24}$$

The consistency condition [15] for our amplitudes is,

$$\Delta^\pi_F(q)\, k_\nu \mathcal{M}^a_{\pi,\,\nu} \,=\, \Delta^\pi_F(q_1)\, i\varepsilon^{3\,a\,b} M^b_\pi\,. \tag{25}$$

Here $\mathcal{M}^a_{\pi,\,\nu}$ is the radiative pion absorption amplitude, M^b_π is the pseudoscalar πNN vertex and $q = k + q_1$.

The amplitudes, which contribute in the leading order, are the nucleon Born terms $M^{B,\,a}_{\mu\nu}(k,q)$ (corresponding to $M(b)$, $M(c)$, $M(d)$ in [12]), the amplitudes $M^a_{\mu\nu}(\pi,1)$, $M^a_{\mu\nu}(\pi,2)$ and

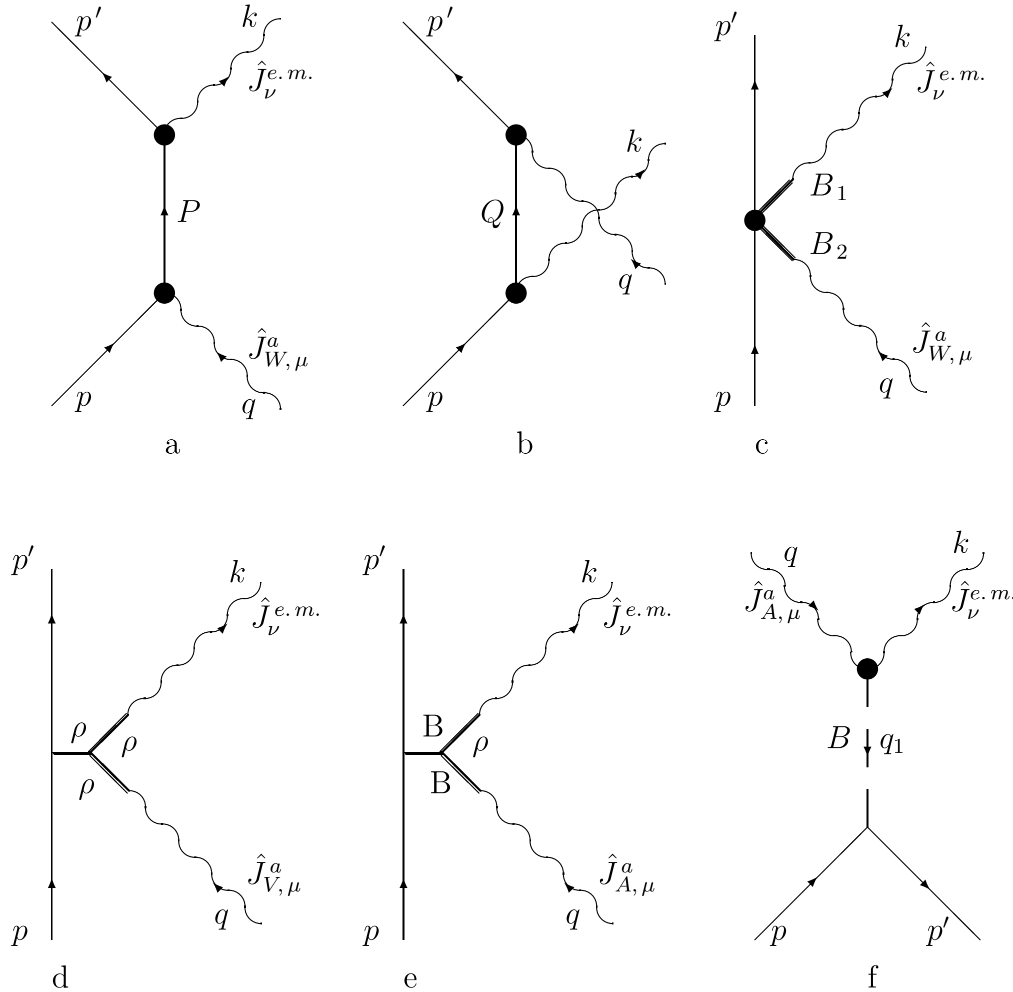

Fig. 1: The radiative hadron amplitude. Graphs a,b – the nucleon Born terms; graphs c,d – the contact terms related to the nucleon Born terms, the possible pairs (B_1, B_2) are (ρ, π), (ω, π) and (ρ, ρ); graph e – mesonic current, B=π or a_1; graph f – the contact terms of the $B = \pi$ or a_1 range.

$M^{B,\,a}_{\mu\nu}(5)$ (the sum of them corresponds to $M(e)$ in [12]), and the mesonic amplitude $M^{m.\,c.,\,a}_{\mu\nu}$ (in correspondence with $M(f)$ in [12]).

The non-relativistic reduction of these amplitudes yields the following contributions to the form factors g_i,

$$
\begin{aligned}
g_1 &= -\lambda_+ g_V^L \left[1 + \frac{\vec{s}}{2M} \cdot \hat{k}\right] - g_V^N \eta + g_A^N \lambda \eta \mu_V - g_M^N \eta \mu_V \,\frac{\vec{\nu}}{2M} \cdot \hat{k}\,, \\
g_2 &= -\lambda_+ g_A^L - g_P^N \eta + g_V^N \lambda \eta \mu_V - g_A^N \eta\,, \\
g_3 &= -\lambda_+ g_A^L + g_M^N \eta + g_V^N \eta - g_A^N \lambda \eta \mu_S\,,
\end{aligned}
$$

$$
\begin{aligned}
g_4 &= \lambda_+ g_A^L - g_V^N \lambda\eta\mu_V + g_P^L\left[\lambda_- + \frac{\nu}{m_\mu}(1-y)\lambda_+\right], \quad g_4' = g_4\frac{k}{2M}, \quad g_4'' = g_4\frac{\nu}{2M}, \\
g_5 &= \lambda_+ g_V^L + g_M^N\eta\mu_V + g_V^N\eta\mu_V + g_M^N\lambda\eta, \quad g_5' = g_5\frac{\nu}{2M}, \\
g_6 &= \lambda_+ g_V^L + (g_P^N + g_A^N)\lambda\eta\mu_V + g_M^N\eta - g_V^N\eta(1+2\mu_n), \quad g_6' = g_6\frac{\nu}{2M}, \\
g_7 &= \lambda_+(g_V^L + g_M^L) + (g_P^N + g_A^N)\lambda\eta\mu_V - g_V^N\eta, \quad g_7' = g_7\frac{k}{2M}, \quad g_7'' = g_7\frac{\nu}{2M}, \\
g_8 &= -\lambda_+(g_V^L + g_M^L) - g_V^N\lambda\eta\mu_S, g_8' = g_8\frac{k}{2M}, \quad g_8'' = g_8\frac{\nu}{2M}, \\
g_9 &= \lambda_+(g_V^L + g_M^L) + (g_V^N + g_M^N)\lambda\eta\mu_S - 2g_A^N\eta\mu_n, \quad g_9' = g_6\frac{\nu}{2M}, \\
g_{10} &= g_P^N\eta\frac{4M\nu}{m_\pi^2 + (q^L)^2} + \lambda_+ g_P^L\frac{\nu}{m_\mu}, \quad g_{10}' = g_{10}\frac{k}{2M}, \quad g_{10}'' = g_{10}\frac{\nu}{2M}, \\
g_{11} &= \lambda_+ g_P^L\frac{\nu}{m_\mu}, \quad g_{11}' = g_{11}\frac{k}{2M}, \quad g_{11}'' = g_{11}\frac{\nu}{2M}.
\end{aligned} \tag{26}
$$

Here our notations mostly follow [12],

$$
\vec{s} = \vec{k} + \vec{\nu}, \quad \eta = \frac{m_\mu}{2M}, \quad \lambda_\pm = \frac{1}{2}(1 \pm \lambda).
$$

In addition we have,

$$
\mu_V = 1 + \mu_p - \mu_n \equiv 1 + \kappa_V, \quad \mu_S = 1 + \mu_p + \mu_S \equiv 1 + \kappa_S. \tag{27}
$$

Besides the obvious momentum dependence,

$$
g_P^L = \frac{2Mm_\mu}{m_\pi^2 + (q^L)^2}\, g_A, \quad g_P^N = \frac{2Mm_\mu}{m_\pi^2 + (q^N)^2}\, g_A, \tag{28}
$$

with

$$
q^L = p - p' = -q_1, \quad q^N = q^L + k,
$$

all other nucleon weak vector and axial–vector form factors are assumed to have the dipole momentum dependence with $m_V = 0.8$ GeV and $m_A = 1$ GeV, respectively.

3.1.1 Next to the leading order corrections

Here we have two groups of contributions. The first one arises from the expansion of the amplitudes considered above by one order more in $1/M$ which leads to,

$$
\begin{aligned}
\left(\frac{2M}{\eta}\right)\Delta g_1 &= g_M^N k - \left(g_V^N\mu_S - g_A^N\lambda\eta\mu_n\right)\left(\vec{s}\cdot\hat{k}\right), \\
\left(\frac{2M}{\eta}\right)\Delta g_2 &= -g_M^N\lambda\left(\vec{\nu}\cdot\hat{k}\right) + 2\mu_n\left(g_A^N + \lambda g_V^N - g_P^N\right)\left(\vec{s}\cdot\hat{k}\right),
\end{aligned}
$$

$$
\begin{aligned}
\left(\frac{2M}{\eta}\right)\Delta g_3 &= -g_M^N k + 2g_A^N \lambda\mu_n \left(\vec{s}\cdot\hat{k}\right) - 2g_V^N \mu_n \left(\vec{\nu}\cdot\hat{k}\right)\,, \\
\left(\frac{2M}{\nu\eta}\right)\Delta g_4'' &= -g_M^N \lambda\mu_S - 2g_V^N \lambda\mu_n\,, \\
\left(\frac{M}{\nu\eta}\right)\Delta g_8'' &= g_V^N \lambda\mu_n\,, \\
\left(\frac{M}{\nu\eta}\right)\Delta g_{10}' &= \left(g_P^N - g_A^N\right)\mu_n\,.
\end{aligned} \tag{29}
$$

Almost entire contribution to the photon spectrum arises from the terms proportional to g_A^N and g_V^N in Δg_2. These terms appear due to the neutron recoil induced by the time component of the weak current.

The second group of corrections of the same order (the HP correction) stems from some contact terms present in the hadron radiative part of the amplitude (22). It is discussed in Sect. 4 of Ref. [36]. Here we quote the results of the non–relativistic reduction,

$$
\begin{aligned}
\Delta g_1 &= -2g_V^L \left(\frac{2M}{m_\rho}\right)^2 \eta\frac{k}{2M}\,, \quad \Delta g_4' = g_A^N \left(\frac{2M}{m_\rho}\right)^2 \eta\frac{k}{2M}\,, \\
\Delta g_2 &= -\frac{g_A^N}{2}\left(\frac{2M}{m_\rho}\right)^2 \eta\frac{\nu+2k}{2M} - \frac{g_P^N}{2}\left(\frac{2M}{m_\rho}\right)^2 \eta\frac{k}{2M}\frac{k+y\nu}{2M}\,, \\
\Delta g_4'' &= -\frac{g_A^N}{2}\left(\frac{2M}{m_\rho}\right)^2 \eta\frac{\nu}{2M}\,, \quad \Delta g_6' = -2g_V^L \left(\frac{2M}{m_\rho}\right)^2 \eta\frac{\nu}{2M}\,, \\
\Delta g_8'' &= -\frac{g_A^N}{2}\left(\frac{2M}{m_\rho}\right)^2 \eta\frac{k}{2M}\,, \quad \Delta g_9' = g_A^N \left(\frac{2M}{m_\rho}\right)^2 \eta\frac{\nu}{2M}\,.
\end{aligned} \tag{30}
$$

3.2 The RMC amplitude with Δ's

We derive the RMC amplitudes arising due to the Δ excitations from chiral Lagrangians [27]–[30]. They correspond to the graphs in Figs. 1a and 1b with the $\Delta(1240)$ instead of nucleon in the intermediate state. The needed Lagrangian reads,

$$
\mathcal{L}^M_{N\Delta\pi\rho a_1} = \frac{f_{\pi N\Delta}}{m_\pi}\,\bar{\Psi}_\mu \vec{T}\mathcal{O}_{\mu\nu}(Z)\Psi\cdot(\partial_\nu\vec{\pi} + 2f_\pi g_\rho \vec{a}_\nu) - g_\rho\frac{G_1}{M}\bar{\Psi}_\mu\vec{T}\mathcal{O}_{\mu\eta}(Y)\gamma_5\gamma_\nu\Psi\cdot\vec{\rho}_{\eta\nu} + h.\,c. \tag{31}
$$

Here $\vec{T}$ is the operator of the transition spin. Another possible term in the $\rho N\Delta$ vertex is suppressed by one order in $1/M$ and it does not contribute in any sizeable manner [20].

We take the operator $\mathcal{O}_{\mu\nu}(B)$ in a form [28]–[30]

$$
\mathcal{O}_{\mu\nu}(B) = \delta_{\mu\nu} + C(B)\,\gamma_\mu\gamma_\nu\,, \tag{32}
$$

$$
C(B) = \frac{1}{2}(1 + 4B)\,A + B\,. \tag{33}
$$

A choice $A=-1$ simplifies considerably [28] the propagator of the Δ.

The coupling constant $f_{\pi N\Delta}$ is not well known and the values for $f^2_{\pi N\Delta}/4\pi$ from the interval between 0.23 and 0.36 can be found in the literature [6]. From the dispersion theory [38], $f^2_{\pi N\Delta}/4\pi \approx 0.30$ and $f^2_{\pi N\Delta}/4\pi \approx 0.35$ from the decay width [39]. Also a good fit to the 33 phase shift was obtained in [29],[30] by using $f^2_{\pi N\Delta}/4\pi \approx 0.314$. The new data on pion photoproduction prefer $f^2_{\pi N\Delta}/4\pi \approx 0.371$ [32]. The ranges of the other relevant parameters of the model are [30],[31],[32],

$$-0.8 \le Z \le 0.7\,, \quad -1.25 \le Y \le 1.75\,, \quad 1.97 \le G_1 \le 2.65\,. \tag{34}$$

Our radiative amplitude with the Δ excitation can be written analogously with the nucleon Born term $M^{B,\,a}_{\mu\nu}(1)$ [3] as,

$$\begin{aligned} M^{\Delta,\,a}_{\mu\nu} \;&=\; -\bar{u}(p')\Big[\left(\hat{J}_{W,\,\mu\alpha}(-q)\right)^+ S^{\alpha\gamma}_F(Q)\hat{J}_{em,\,\nu\gamma}(k)\,\left(T^+\right)^a T^3 \\ &\qquad + \left(\hat{J}_{em,\,\nu\gamma}(-k)\right)^+ S^{\gamma\alpha}_F(P)\hat{J}_{W,\,\mu\alpha}(q)\,\left(T^+\right)^3 T^a\Big]\,u(p)\,, \end{aligned} \tag{35}$$

Here the weak $N\Delta$ vertex reads,

$$\hat{J}_{W,\,\mu\alpha}(q) = \hat{J}_{V,\,\mu\alpha}(q) - \hat{J}_{A,\,\mu\alpha}(q)\,, \tag{36}$$

with the vector part defined as,

$$\hat{J}_{V,\,\mu\alpha}(q) = i\left(\frac{G_1}{M}\right) m^2_\rho \Delta^\rho_F(q)\,(q_\beta\delta_{\mu\lambda} - q_\lambda\delta_{\mu\beta})\;\mathcal{O}_{\alpha\beta}(Y)\,\gamma_5\gamma_\lambda\,, \tag{37}$$

and with the axial–vector part of the form,

$$\hat{J}_{A,\,\mu\alpha}(q) = \left(\frac{f_\pi f_{\pi N\Delta}}{m_\pi}\right)\left[m^2_{a_1}\Delta^{a_1}_{\mu\lambda}(q) - q_\mu q_\lambda \Delta^\pi_F(q)\right]\,\mathcal{O}_{\alpha\lambda}(Z)\,. \tag{38}$$

Further the electromagnetic $\gamma N\Delta$ vertex is,

$$\hat{J}_{em,\,\nu\gamma}(k) = -\hat{J}_{V,\,\nu\gamma}(k,\,k^2=0) = -i\left(\frac{G_1}{M}\right)(k_\beta\delta_{\nu\lambda} - k_\lambda\delta_{\nu\beta})\;\mathcal{O}_{\gamma\beta}(Y)\,\gamma_5\gamma_\lambda\,. \tag{39}$$

At last, $S^{\alpha\gamma}_F(p)$ is the Δ isobar propagator. With the choice $\mathcal{O}_{\alpha\beta} = \delta_{\alpha\beta}$, our amplitudes (36)–(38) coincide in the form with the ones obtained in [20] from the study of the weak N–Δ vertex in the reaction $\nu\, d \,\to\, \mu^-\,\Delta^{++}\,n$ [‡].

For the divergence of the resonant amplitude $M^{\Delta,\,a}_{\mu\nu}$ from Eq. (35) we have,

$$q_\mu M^{\Delta,\,a}_{\mu\nu} \;=\; if_\pi m^2_\pi \Delta^\pi_F(q^2)\,M^{\Delta,\,a}_{\pi,\,\nu}\,. \tag{40}$$

[‡]For a recent study of this reaction see [40].

Here the associated resonant radiative pion absorption amplitude $M^{\Delta,\,a}_{\pi,\,\nu}$ is,

$$\begin{aligned} M^{\Delta,\,a}_{\pi,\,\nu} &= -\bar{u}(p')\left[\left(\hat{M}^{\Delta}_{\pi,\,\alpha}(-q)\right)^{+} S^{\alpha\gamma}_{F}(Q)\hat{J}_{em,\,\nu\gamma}(k)\,\left(T^{+}\right)^{a} T^{3}\right. \\ &\qquad \left. +\left(\hat{J}_{em,\,\nu\gamma}(-k)\right)^{+} S^{\gamma\alpha}_{F}(P)\hat{M}^{\Delta}_{\pi,\,\alpha}(q)\,\left(T^{+}\right)^{3} T^{a}\right] u(p)\,, \end{aligned} \tag{41}$$

and the $\pi N\Delta$ vertex reads,

$$\hat{M}^{\Delta}_{\pi,\,\alpha}(q) = i\frac{f_{\pi N\Delta}}{m_\pi} q_\lambda\, \mathcal{O}_{\alpha\lambda}(Z)\,. \tag{42}$$

We now present the contributions from the amplitude (35) to the form factors g_i. They are,

$$\begin{aligned} \Delta g_1 &= \frac{2}{3}\lambda(C_- - C_+)C\left\{1+(1-R)\left[C(Y)+C(Z)+2(2+R)C(Y)C(Z)\right]\right\}\,, \\ \Delta g_2 &= \frac{1}{3}(C^+ + C_-)C(1-R)\left[-(1+2R)+2(1-2R)C(Y)+2(1-R)C(Z)\right. \\ &\quad \left.+4(2-R)C(Y)C(Z)\right] + \frac{1}{3}(C^+ + C_-)C\frac{g^N_P}{2Mg^N_A}\left\langle -\left[(1-R)(1+2R)k\right.\right. \\ &\quad \left.+y\nu\right] + 2(1-R)\left\{\left[(1-2R)k+y\nu\right]C(Y)+\left[(1-R)k+y\nu\right]C(Z)\right. \\ &\quad \left.\left. +2\left[(2-R)k+(2+R)y\nu\right]C(Y)C(Z)\right\}\right\rangle\,, \\ \Delta g_3 &= \frac{2}{3}\lambda(C_+ + C_-)C\left\{1+(1-R)\left[C(Y)+C(Z)+2(2+R)C(Y)C(Z)\right]\right\}\,, \\ \Delta g'_4 &= -\Delta g''_8 \,=\, (C_+ + C_-)C\,, \\ \Delta g_6 &= -\lambda\frac{\nu}{3M}\frac{g^N_P}{g_A}(C_+ - C_-)C\left\{1+(1-R)\left[C(Y)+C(Z)+2(2+R)C(Y)C(Z)\right]\right\}\,, \\ \Delta g'_{10} &= (C_+ + C_-)C\frac{\nu}{6M}\frac{g^N_P}{g_A}\left\{1-2(1-R)\left[C(Y)+C(Z)+2(2+R)C(Y)C(Z)\right]\right\}\,, \end{aligned} \tag{43}$$

where

$$\begin{aligned} C &= -\frac{4}{3}\frac{f_\pi f_{\pi N\Delta}}{m_\pi}G_1\eta k\,, \quad R \,=\, M/M_\Delta\,, \\ C^{-1}_{+} &= \left\{(M_\Delta - M) + \frac{2M}{M_\Delta + M}\left[m_\mu - \nu + \frac{m_\mu}{2M}\left(2\nu - m_\mu - \frac{2\nu}{m_\mu}(\nu + yk)\right)\right]\right\}\,, \\ C^{-1}_{-} &= \left\{(M_\Delta - M) + \frac{2M}{M_\Delta + M}\left[\nu - m_\mu + \frac{m_\mu}{2M}\left(2\nu - m_\mu\right)\right]\right\}\,. \end{aligned} \tag{44}$$

According to the concept developed in [28]–[30], we take the mass of the Δ isobar real.

3.3 The RMC amplitude from an anomalous Lagrangian of the $\pi\rho\omega a_1$ system

We have considered so far the amplitudes where a natural parity does not change in any vertex. The natural parity of a particle is defined as $P\,(-1)^J$, where P is the intrinsic parity and J is the spin of the particle. Some amplitudes of this kind relevant for the process under study are presented in Fig. 2.

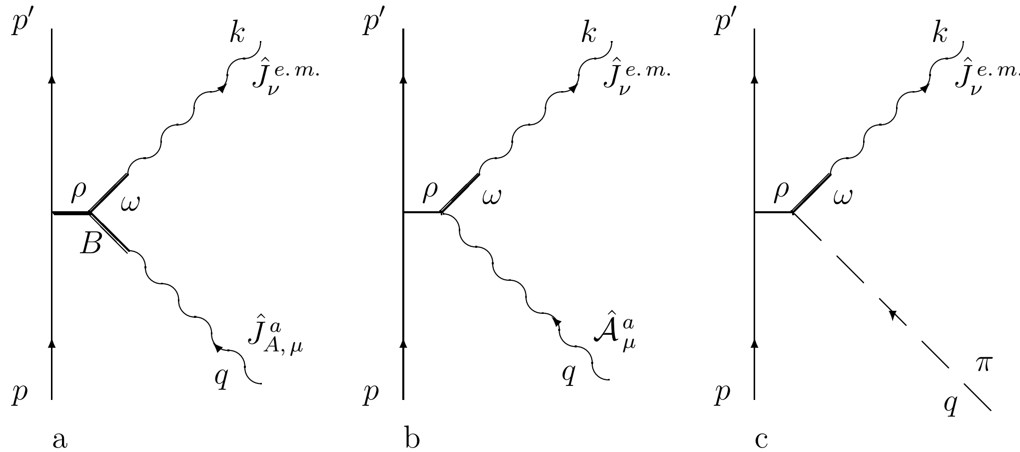

Fig. 2: The radiative hadron amplitudes from an anomalous Lagrangian of the $\pi\rho\omega a_1$ system. In graph a, B=π or a_1; graph c – the associated radiative pion absorption amplitude. These amplitudes satisfy the PCAC equation.

The starting point is an anomalous Lagrangian of the $\pi\rho\omega a_1$ system [36],[37] constructed within the approach of hidden local symmetries [41],[42]. The electromagnetic interaction in such a system was first considered in [43] and the relevant constants $\tilde{c}_i$ were extracted from the data as well. The weak interaction was incorporated explicitly in [36] and the refit of the constants to the modern data [44] was made in [37].

The Lagrangian reads,

$$\begin{aligned}
\mathcal{L}_{an} &= 2ig_\rho \varepsilon_{\kappa\lambda\mu\nu} \Big[(\partial_\kappa \omega_\lambda)\left(g_\rho \vec{\rho}_\mu - e\vec{\mathcal{V}}_\mu\right) + \left(g_\rho \omega_\kappa - \frac{1}{3} e\,\mathcal{B}_\kappa\right)(\partial_\lambda \vec{\rho}_\mu)\Big] \\
&\quad \cdot \left[\tilde{c}_7 \left(\frac{1}{f_\pi}\,\partial_\nu \vec{\pi} + e\vec{\mathcal{A}}_\nu\right) + \tilde{c}_8 \left(\frac{1}{2} e\vec{\mathcal{A}}_\nu) - g_\rho \vec{a}_\nu\right)\right] \\
&\quad + 2ie\varepsilon_{\kappa\lambda\mu\nu} \left[\left(\frac{1}{3}\partial_\kappa \mathcal{B}_\lambda\right)\left(g_\rho \vec{\rho}_\mu - e\vec{\mathcal{V}}_\mu\right) + \left(g_\rho \omega_\kappa - \frac{1}{3} e\,\mathcal{B}_\kappa\right)\left(\partial_\lambda \vec{\mathcal{V}}_\mu\right)\right] \\
&\quad \cdot \left[\tilde{c}_9 \left(\frac{1}{f_\pi}\,\partial_\nu \vec{\pi} + e\vec{\mathcal{A}}_\nu\right) + \tilde{c}_{10} \left(\frac{1}{2} e\vec{\mathcal{A}}_\nu) - g_\rho \vec{a}_\nu\right)\right] ,
\end{aligned} \tag{45}$$

where besides the meson fields, the external vector isoscalar $\mathcal{B}_\mu$ and isovector $\vec{\mathcal{V}}_\mu$ and axial vector isovector $\vec{\mathcal{A}}_\mu$ fields are also included. The constants $\tilde{c}_i$ are [37],

$$\begin{aligned}
\tilde{c}_7 &= 8.64 \times 10^{-3} \quad & \tilde{c}_8 &= -1.02 \times 10^{-1} , \\
\tilde{c}_9 &= 9.23 \times 10^{-3} \quad & \tilde{c}_{10} &= 1.29 \times 10^{-1} .
\end{aligned} \tag{46}$$

The RMC amplitudes arising from the anomalous Lagrangian (45) are,

$$
\begin{aligned}
M^{an,\,a}_{\mu\nu}(1) &= i\frac{g_\rho^2}{3}\,\varepsilon_{\eta\nu\beta\sigma}\,k_\eta q_\sigma q_\mu \Delta^\pi_F(q)\,\Delta^\rho_{\beta\lambda}(q_1)\bar{u}(p')\left(\gamma_\lambda\,-\,\frac{\kappa_V}{2M}\sigma_{\lambda\alpha}q_{1\,\alpha}\right)\tau^a u(p)\,,\\
M^{an,\,a}_{\mu\nu}(2) &= -i\frac{g_\rho^2}{3}\,\varepsilon_{\eta\nu\beta\mu}\,k_\eta\,\Delta^\rho_{\beta\lambda}(q_1)\bar{u}(p')\left(\gamma_\lambda\,-\,\frac{\kappa_V}{2M}\sigma_{\lambda\alpha}q_{1\,\alpha}\right)\tau^a u(p)\,,
\end{aligned}
\tag{47}
$$

and they correspond to the processes presented in Fig. 2a and Fig. 2b. Together with the radiative pion absorption amplitude of Fig. 2c,

$$
M^{an,\,a}_{\pi,\,\nu} \,=\, -\frac{g_\rho}{3f_\pi}\,\varepsilon_{\eta\nu\beta\sigma}\,k_\eta q_\sigma \Delta^\rho_{\beta\lambda}(q_1)\bar{u}(p')\left(\gamma_\lambda\,-\,\frac{\kappa_V}{2M}\sigma_{\lambda\alpha}q_{1\,\alpha}\right)\tau^a u(p)\,,
\tag{48}
$$

the amplitudes (47) satisfy the PCAC equation,

$$
q_\mu\left[M^{an,\,a}_{\mu\nu}(1)\,+\,M^{an,\,a}_{\mu\nu}(2)\right] \,=\, if_\pi m_\pi^2 \Delta^\pi_F(q)\,M^{an,\,a}_{\pi,\,\nu}\,.
\tag{49}
$$

The contribution from the anomalous amplitudes (47) to the total hadron radiative amplitude (22) for the reaction (16) is given by,

$$
T^a_{an} \,=\, \frac{eG}{\sqrt{2}} l_\mu(0)\varepsilon_\nu\;(\tilde{c}_7+\tilde{c}_9)\left[M^{an,\,a}_{\mu\nu}(1)\,+\,M^{an,\,a}_{\mu\nu}(2)\right]\,.
\tag{50}
$$

Actually, the contact amplitude $M^{an,\,a}_{\mu\nu}(2)$ prevails and we present the contribution from it to the form factors g_i as,

$$
\begin{aligned}
\Delta g_2 &= \frac{g_\rho^2}{3}\left(\frac{2M}{m_\rho}\right)^2(1+\kappa_V)\eta\,\frac{\vec{k}\cdot\vec{s}}{(2M)^2}\,,\\
\Delta g_8' &= -\frac{g_\rho^2}{3}\left(\frac{2M}{m_\rho}\right)^2(1+\kappa_V)\eta\,\frac{k\nu}{(2M)^2}\,,\\
\Delta g_8'' &= -\frac{g_\rho^2}{3}\left(\frac{2M}{m_\rho}\right)^2(1+\kappa_V)\eta\,\frac{k^2}{(2M)^2}\,,\\
\Delta g_9' &= -\Delta g_{10}' \,=\, \Delta g_8'\,.
\end{aligned}
\tag{51}
$$

One can obtain more general result for Δg_i (51) by a change

$$
1/3\,\rightarrow\,g_{\rho 1}/g_{\omega 1}\,,\quad \kappa_V\,\rightarrow\,g_{\rho 2}/g_{\rho 1}\,,
$$

which coresponds to the model used in [29],[30] for describing the pion photoproduction amplitude in the t channel. In this model, the $\pi\rho\gamma$ and $\pi\omega\gamma$ amplitudes are effectively the same as those obtained from our anomalous Lagrangian (45) and the ρNN and ωNN vertices contain four constants $g_{\rho 1}$, $g_{\rho 2}$, $g_{\omega 1}$ and $g_{\omega 2}$, which are the free parameters, obtained together with other free parameters of the model from the fit to the data.

Compared with the sets (29) and (30), the g_i's (51) are even larger. However, due to the values (46) of $\tilde{c}_i$, the factor $(\tilde{c}_7+\tilde{c}_9)\,\approx\,1.8\times 10^{-2}$ makes the contribution from the amplitude T^a_{an} (50) negligeably small.

4 Results and conclusion

Using the Hamiltonian (19) and the sets of the form factors g_i (26), (29),(30) and (43), we have calculated the photon energy spectra for the radiative muon capture in a muon-hydrogen system described by a spin density matrix ρ [45],

$$\begin{aligned}\frac{dw_{fi}}{dk} &= \frac{1}{4\pi^3}\left(\alpha^2 G_F \cos\theta_c\, m/m_\mu\right)^2 m M_n k \int_{-1}^{+1} dy \frac{\nu_0^2}{W+k(y-1)}\frac{1}{4} Tr\left\{(1-\vec{\sigma}\cdot\hat{\nu})\right. \\ &\left.\times H_{eff}^{(0)}\,\rho\,\left[H_{eff}^{(0)}\right]^+\right\}. \qquad (52)\end{aligned}$$

Here α is the fine structure constant, G_F is the Fermi constant, $\cos\theta_c$ is the Cabibbo angle, m is the reduced mass of the $\mu - p$ system,

$$\nu_0 = \frac{W(k_{max}-k)}{W+k(y-1)}, \quad k_{max} = \frac{W^2-M_n^2}{2W}, \quad W = M_p + m_\mu,$$

and $M_{p(n)}$ is the proton (neutron) mass.

The mixture of muonic states relevant to the TRIUMF experiment [21],[22] is 6.1 % in the singlet μp state, 85.4 % in the ortho pμp state and 8.5 % in the para pμp state, for which we present our results in Fig. 3 and Fig. 4.

The curves present, in percentage, the ratio of (the effect with a correction - the effect without a correction) to the effect with a correction. In Fig. 3, we present by the dotted curve the change in the leading order spectrum [arising from the form factors (26)] due to the first group of the next to the leading order corrections (29), yielding a negative contribution in the range of $\approx$ -1– -11%. On the other side, the sum of the contributions from the leading order terms (26) and from the form factors (29) should well reproduce the photon spectrum obtained in the relativistic calculations [16] without Δ's. We shall call the corresponding spectrum the basic (B) one.

The second group of the form factors (30) (the HP correction) yields an enhancement of $\approx$ 2–4% (dashed curve) relative to the spectrum B, which partially compensates the negative effect from the first group of corrections (29).

In order to estimate the model dependence of the Δ excitation effect, we calculated the spectra using various sets of the parameters Y, $f_{\pi N\Delta}$ and G_1 allowed by the photoproduction data [29]–[32]. We have found that the spectra do not practically depend on the parameter Z in a rather wide range of its values. The set

$$g_P = g_P^{PCAC}, \quad G_1 = 2.4, \quad f_{\pi N\Delta}^2/4\pi = 0.314, \quad Y = 1.75, \quad Z = -0.25, \qquad (53)$$

provides the dot–dashed curve in Fig. 3, which represents the combined HP and the Δ isobar effect relative to the B spectrum. Analogously, the solid curve in Fig. 3 corresponds to the set,

$$g_P = g_P^{PCAC}, \quad G_1 = 2.65, \quad f_{\pi N\Delta}^2/4\pi = 0.371, \quad Y = -1.25, \quad Z = -0.25. \qquad (54)$$

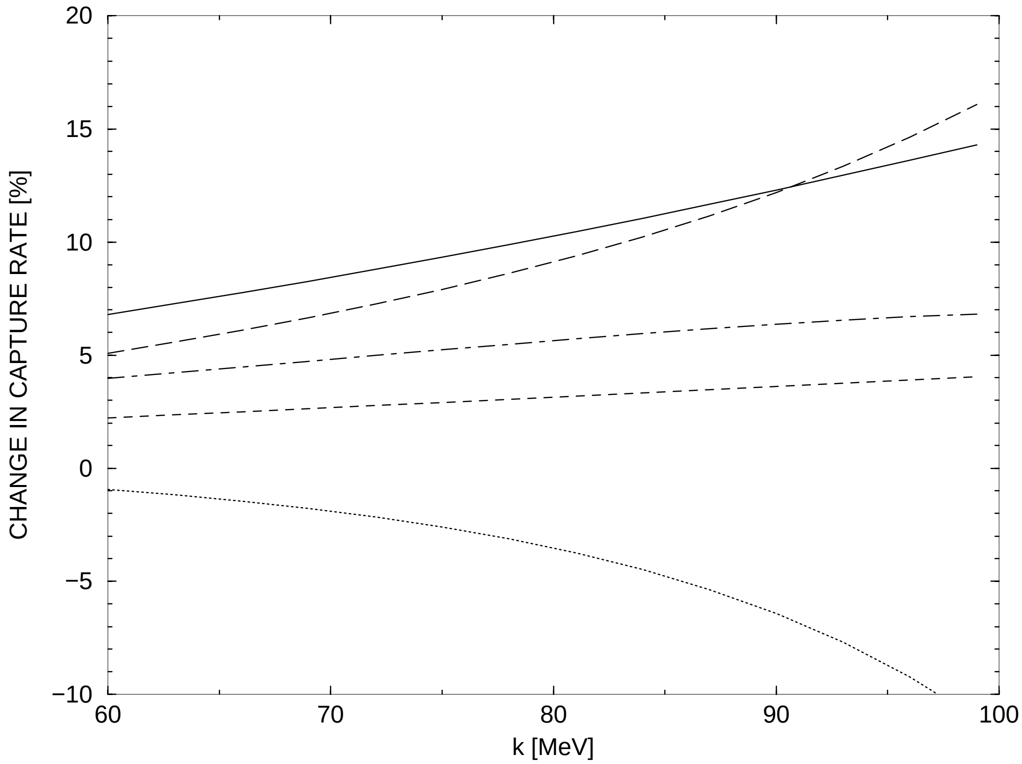

Fig. 3: Photon energy spectra calculated for the mixture of muonic states relevant to the TRIUMF experiment [21],[22]. At the vertical axis, we present the difference in % for various spectra (see text).

This curve represents the maximal combined effect from the HP corrections and from the Δ isobar excitation process in the allowed range of the values of the parameters Y, Z, $f_{\pi N\Delta}$ and G_1 discussed in Sect. 3.2.

The long–dashed curve in Fig. 3 shows the change of the B spectrum, if the value of g_P (9) is changed by 25 %. This curve is quite close to the solid one which means that this model can explain up to half of the discrepancy between the g_P^{PCAC} (9) and of g_P extracted from the experiment [21],[22], if one restricts oneself to the set of the parameters from Eqs. (34),(53) and (54). They were extracted in [29]–[32] from the data on pion photoproduction by using unitarized multipoles arising from the pion photoproduction amplitude. The problem with the unitarity appears in the pion photoproduction, because of the pion–nucleon interaction in the final state. It is true that the time reversed of the pion production amplitude is connected with the hadron radiative part of our RMC amplitude by the continuity equations (23),(40) and (49). However, we do not need to unitarize our amplitude. Therefore it is not clear, how the inequalities (34) are restrictive for the problem considered here.

The correct way to do in the given case would be to extract the parameters of the model, G_1, $f_{\pi N\Delta}$, Y and g_P[§], from the fit to the experimental photon spectrum [21],[22]. Here we present in Fig. 4 the dependence of the change of the spectra on the parameter Y. The

[§]As we have already mentioned above, the calculations are insensitive to the variation of Z.

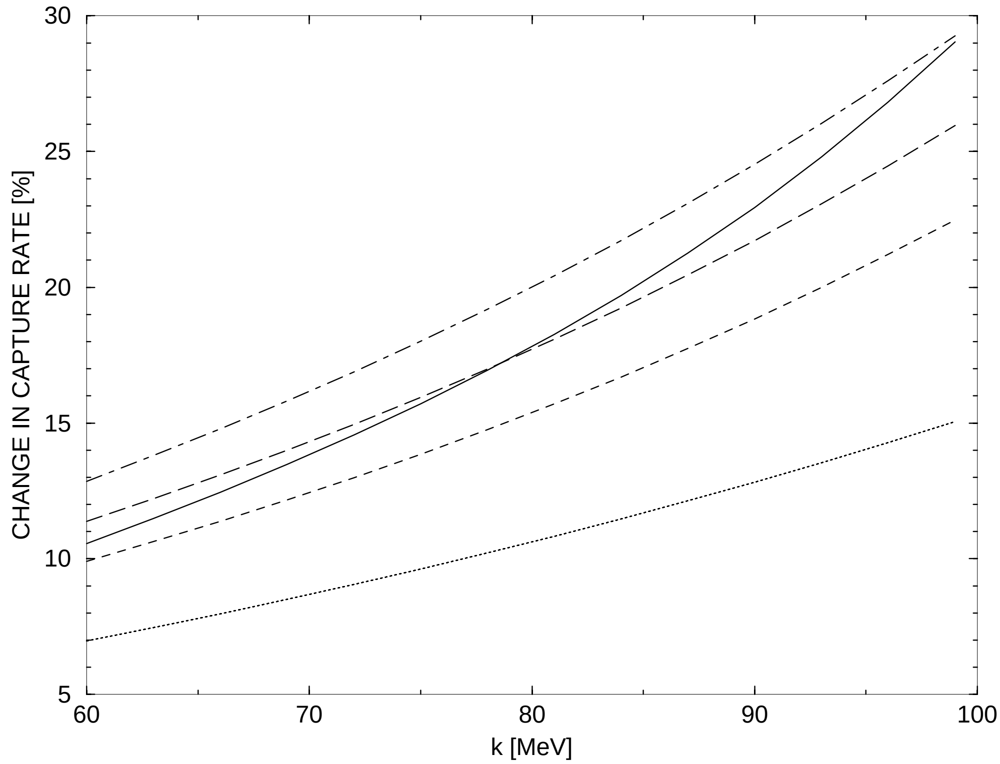

Fig. 4: The dependence of the photon energy spectra on the parameter Y characterizing the off–shellness of the Δ isobar (see text).

dotted, dashed, long–dashed and dot–dashed curves correspond to the values Y=-2.25, -6.25, -8.25 and -10.25, respectively. The values of the other parameters are taken as,

$$g_P \,=\, g_P^{PCAC}\,, \quad G_1 \,=\, 2.525\,, \quad f^2_{\pi N\Delta}/4\pi \,=\, 0.314\,, \quad Z \,=\, -0.25\,. \tag{55}$$

The solid curve shows the change of the B spectrum, if the value of the g_P (9) is changed by 50 %, which should correspond to what is observed in the experiment. It is seen that the model can reproduce reasonably the experimental photon spectrum [21],[22] for the PCAC value (9) of g_P, if the value of the parameter Y is taken $Y \approx -9$ and other parameters of the model are from Eq. (55). However we realize that only a careful analysis of the data can prove the validity of the model.

In this paper, we have presented the photon energy spectra for the RMC in hydrogen, calculated using the effective Hamiltonian (19), where the form factors g_i were derived from the chiral Lagrangian of the $N\Delta\pi\rho\omega a_1$ system. The non–resonant part of the Lagrangian contains the normal and anomalous Lagrangians of the $N\pi\rho\omega$ system interacting with the external electromagnetic and weak fields by the associated one–body currents. For the resonant part of our Lagrangian we have extended our model by adopting results of the model developed by Olsson and Osypowski [28] and by Davidson, Mukhopadhyay and Wittman [29],[30]. The calculated combined effect, due to the Δ isobar excitation correction (43) and the HP correction (51), can explain up to half of the discrepancy between the PCAC value of the g_P (9) and of g_P extracted from the experiment [21],[22], if one restricts oneself to the values of the parameters of the model extracted from the data on the pion production by the electromagnetic interaction off the nucleon. If one is allowed to vary the parameters of the model independently, the experimental photon spectrum can be described for the values of the induced pseudoscalar *constant, $g_P \approx g_P^{PCAC}$, reasonably well.

Acknowledgments

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